

9/13/21

## MATH 323-01

## 12.6 - Quadratic Surfaces

Ex: Understand the surface w/ equation

$$x^2 - y^2 - z^2 - 4x - 2z + 3 = 0.$$

Sol: First rewrite the equation  
(via Completing the Square):

$$0 = x^2 - y^2 - z^2 - 4x - 2z + 3$$

$$(a+b)^2$$

$$= a^2 + 2ab + b^2$$

if and only if

$$0 = (x^2 - 2 \cdot 2x + 2^2 - 2^2) + (-y^2) - (z^2 + 2 \cdot 1z + 1^2 - 1^2) + 3$$

$$\text{iff } 0 = (x-2)^2 - y^2 - (z+1)^2 - (-1^2) + 3$$

$$\text{iff } 0 = (x-2)^2 - y^2 - (z+1)^2$$

Now, let's understand the cross-sections of this  
picture with respect to (wrt) the coordinate plane  
(shifted).

← Konstant  
When  $z = k$ :

$$0 = (x-2)^2 - y^2 - (k+1)^2$$

$$\text{i.e. } (x-2)^2 - y^2 = (k+1)^2 \leftarrow \text{Hyperbola}$$

Conic SectionsEllipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

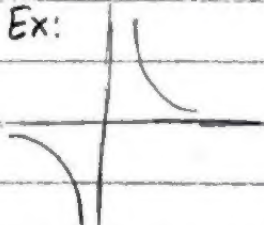
Hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = c$$

Parabola:

$$\frac{x^2}{a^2} - \frac{y}{b} = c$$

Ex:

when  $y = k$ :

$$0 = (x-2)^2 - k^2 - (z+1)^2$$

$$\text{i.e. } (x-2)^2 - (z+1)^2 = k^2$$

Hyperbola

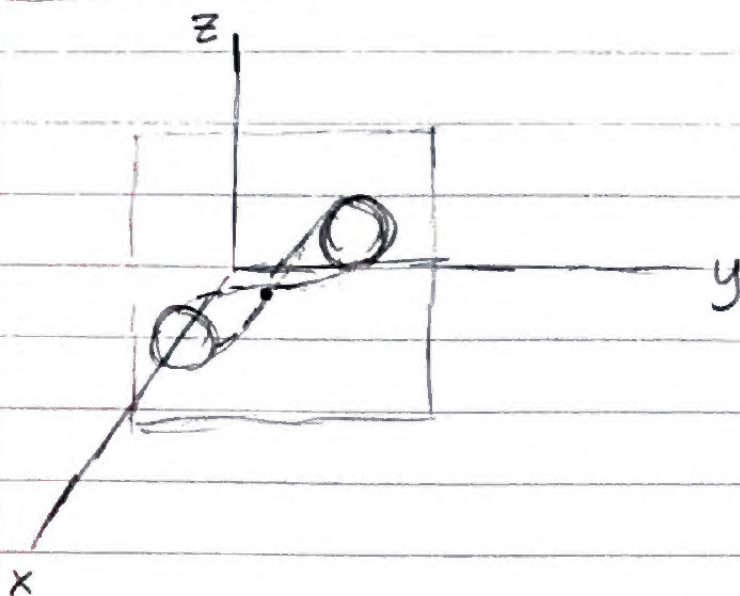
when  $x = k$ :

$$0 = (k-2)^2 - y^2 - (z+1)^2$$

$$\text{i.e. } y^2 + (z+1)^2 = (k-2)^2$$

Ellipse  
(circle or point  
at  $k=2$ )

So a picture:



Cone:

$$\frac{-x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$$

Equation	Name	$x=k$	$y=k$	$z=k$
$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	Ellipsoid	Ellipse (general)	Ellipse (general)	Ellipse (general)
$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z}{c} = 0$	Elliptic Paraboloid	Parabola	Parabola	Ellipse
$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z}{c} = 0$	Hyperbolic Paraboloid	Parabola	Parabola	Hyperbola
$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$	One-sheet Hyperboloid	Hyperbola	Hyperbola	Ellipse
$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$	Cone	Hyperbola	Hyperbola	Ellipse (or point)
$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	Two-sheet Hyperboloid	Hyperbola	Hyperbola	Ellipse (or nothing)



### 13.1 - Space Curves

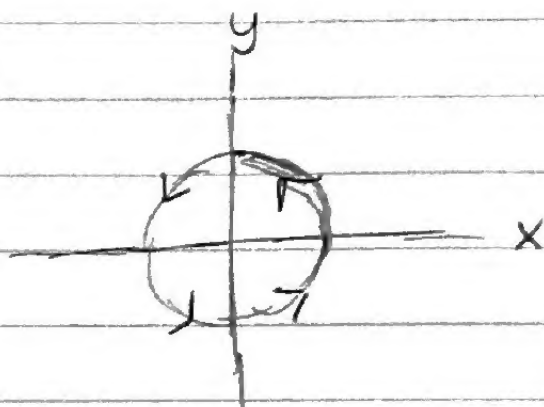
- A space curve is a function:

$$\vec{r} : I \rightarrow \mathbb{R}^n$$

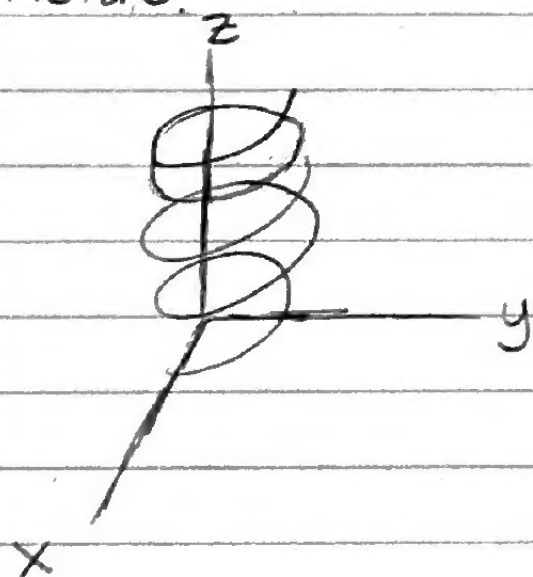
Ex: The helix is the space curve

$$\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle.$$

Shadow in xy-plane :



Picture:



Defn : The limit of a space curve

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$  at "time"  
 $t=a$  is the componentwise limit if they all  
exist, i.e.

$$\begin{aligned} \lim_{t \rightarrow a} \vec{r}(t) &= \lim_{t \rightarrow a} \langle x(t), y(t), z(t) \rangle \\ &= \left\langle \lim_{t \rightarrow a} x(t), \lim_{t \rightarrow a} y(t), \lim_{t \rightarrow a} z(t) \right\rangle \end{aligned}$$

Exercise: Compute the limit of

$$\vec{r}(t) = \langle (4 + \sin(20t))\cos(t), (4 + \sin(20t))\sin(t), \cos(20t) \rangle$$

at  $t = \frac{6\pi}{8}$